# LXXIV. *An Explanation of an Inſtrument for meaſuring ſmall Angles, the firſt Account of which was read before the* Royal Society May 10, 1753. *By Mr.* John Dollond. *In a Letter to* James Short, *M. A. and F. R. S.*

SIR,

Read Apr. 25, 1754. THE account which I gave you, ſome time ago, of a new micrometer, was contained in as few words as poſſible; being rather deſirous, that experiments might be made, before I ſaid much concerning it: But ſince your many repeated experiments have confirmed what was expected from it, I have endeavoured to draw up a more full account of this inſtrument, with demonſtrations of the principles which it is founded upon, which I here ſend you inclos'd, and which you may lay before the Royal Society, if you think proper. I am, Sir,

Your moſt obedient humble ſervant,

Denmark-Court, April 4. 1754. John Dollond.

BEfore I enter upon particulars relating to this micrometer, it will be proper to make a few preparatory obſervations on the nature of ſpherical glaſſes, ſo far as may be neceſſary to render the following explanation more eaſily underſtood.

*Obſervation* 1. It is a property of all convex ſpherical glaſſes to refract the rays of light, which are tranſmitted through them, in ſuch a manner, as to collect all thoſe that proceed diverging from any one point

point of a luminous object, to ſome other point; whoſe diſtance from the glaſs depends chiefly on its convexity, and the diſtance of the object from it.

*Obſ.* 2. The point, where the rays are thus collected, may be conſidered as the image of that point, from which they diverge. For if we conceive ſeveral radiant points thus emitting rays, which, by the refractive quality of the glaſs, are made to converge to as many other points; it will be an eaſy matter to underſtand, how every part of the object will be truly repreſented. As this property of ſpherical glaſſes is explained and demonſtrated by all the writers on optics; it being the very foundation of the ſcience, the bare mention of it is ſufficient for the preſent purpoſe.

*Obſ.* 3. It will be neceſſary, however, to obſerve farther, that the lines connecting every point in the object with its correſponding ones in the image, do all interſect in a certain point of the axis or line paſſing thro' the poles of the glaſs, where its two ſurfaces are parallel, and may be properly called its centre: Whence it appears, that the angles ſubtended by the object and its image from that point, muſt be equal; And therefore their diameters will be in the ſame ratio, as their diſtances from that point.

*Obſ.* 4. As the formation of the image by the glaſs depends entirely on the property above-mentioned, that is to ſay, its collecting all the light, that is incident on it, from the ſeveral points of the object into as many other points at its focus; it follows, that any ſegment of ſuch a glaſs will alſo form an image equal, and every way ſimilar, to that exhibited by the whole glaſs; with this difference only, that it will be ſo much darker, as the area of the ſegment is leſs than that of the whole glaſs.

*Obſ.*

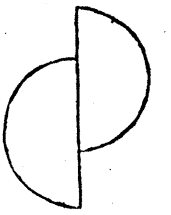
*Obſ.* 5. The axis of a ſpherical glaſs is a line connecting the centres of the ſpheres, to which the two ſurfaces are ground; and where-ever this line paſſes thro' the glaſs, there the ſurfaces are parallel. But if it happens, that this line does not go thro' the ſubſtance of the glaſs, ſuch a glaſs is ſaid to have no internal centre; but it is conceived to be in its plane produced, till it meets the axis: And this imaginary point, tho' external to the glaſs, is as truly its centre, and is as fixed in its poſition to it, as if it were actually within its ſubſtance.

*Obſ.* 6. If a ſpherical glaſs, having its center or pole near its middle or centre of its circumference, ſhould be divided by a ſtrait line through the middle; the centre will be in one of the ſegments only. For how exact ſoever a perſon may be ſuppoſed to be in cutting it thro' the centre; yet 'tis hard to conceive, how a mathematical point ſhould be divided in two: Therefore the centre will be internal to one of the ſegments, and external to the other. But if a ſmall matter be ground away from the ſtrait edge of each ſegment, both their centres will become external; and ſo they will more eaſily be brought to a coincidence.

*Obſ.* 7. If theſe two ſegments ſhould be held together, ſo as to make their centres coincide; the images, which they give of any object, will likewiſe coincide, and become a ſingle one. This will be the caſe, when their ſtrait edges are joined to make the glaſs, as it were, whole again: But let the centres be any-how ſeparated, their images will alſo ſeparate, and each ſegment give a ſeparate and diſtinct image of any object, to which they may be expos'd.

*Obſ.* 8. Tho' the centres of the ſegments may be drawn from their coincidence, by removing the ſeg-

ments in any direction whatever; yet the moſt convenient way for this purpoſe is, to ſlide their ſtrait edges one along the other, till they are remov'd, as the figure in the margin repreſents them: For thus they may be moved without ſuffering any falſe light to come in between them. And by this way of removing them, the diſtance between their centres may be very conveniently meaſured; *viz.* by having a Vernier's diviſion, commonly, tho' falſely, call'd a Nonnius's, fixed to the braſs-work, that holds one ſegment, ſo as to ſlide along a ſcale on the plate, to which the other part of the glaſs is fitted.



*Obſ.* 9. As the images of the ſame object are ſeparated, by the motion of the ſegments; ſo thoſe of different objects, or different parts of the ſame object, may be made to coincide. Suppoſe the ſun, moon, or any planet, to be the object; the two images thereof may, by this contrivance, be removed, till their oppoſite edges are in contact: In which caſe, the diſtance between the centres of the two images will be equal to the diameter of either; and ſo of any other object whatever.

*Obſ.* 10. This divided glaſs may be uſed, as a micrometer, three different ways. In the firſt place, it may be fixed at the end of a tube, of a ſuitable length to its focal diſtance, as an object-glaſs; the other end of the tube having an eye-glaſs fitted as uſual in aſtronomical teleſcopes. Secondly, It may be applied to the end of a tube much ſhorter than its focal diſtance, by having another convex glaſs within the tube, to ſhorten the focal diſtance of that, which is cut in two. Laſtly, It may be applied to the open end of a reflecting

flecting teleſcope; either of the Newtonian, Gregorian, or Caſſegrain conſtruction. And tho' this laſt method is much the beſt, and moſt convenient, of the three; yet, as the firſt is the moſt natural, as well as the eaſieſt to be underſtood; it will be proper to explain it fully, and to demonſtrate the principles, on which this micrometer is conſtructed, by ſuppoſing it made uſe of in the firſt way: Which being done, the application of it to other methods will be readily underſtood.

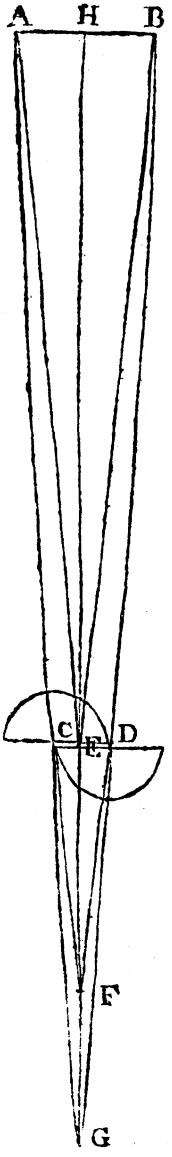
Having thus, by the foregoing obſervations, given a general idea of the nature and effects of this divided object glaſs, I ſhall proceed to demonſtrate the principles, from whence the meaſures of the angles are to be obtained by this inſtrument; which will be done by the following propoſitions.

## PROPOSITION I.

*Suppoſe a divided object-glaſs fixed at the end of a tube, according to the firſt method, and the tube directed to the object intended to be meaſured; and ſuppoſe, likewiſe, the ſegments removed from their original poſition, in the manner directed under* Obſ. 8. *till the oppoſite edges of the two images are ſeen in contact at the focus of the eye-glaſs: Then, I ſay, the angle ſubtended, by the diſtance between the centres of the ſegments, from the focus of the eye-glaſs, where the edges are ſeen in contact, is equal to the angle ſubtended by the diameter of the object from that ſame point.*

### *Demonſtration.*

Let the line *AB* (ſee the next page) repreſent the diameter of the object to be meaſured; and the




points *C D* the centres of the two glaſs ſegments: Alſo *G* the focus where the images of the extremities of the object are coincident. It is evident, from *Obſ.* 3. that *A G* and *B G* are ſtrait lines, that paſs thro' the centres of the ſegments, and connect the extreme points of the object with their correſponding points in the images; and therefore, as the diameter of the object and the diſtance between the centres of the ſegments are both inſcribed between theſe two lines, they muſt needs ſubtend the ſame angle from the point where thoſe lines meet; which is at *G*. *Q. E. D.*

The focal diſtance *C G*, or *D G*, is variable, according to the diſtance of the object from the glaſs: So that it decreaſes as the diſtance of the object from the glaſs increaſes; and when the object is ſo far off, that the focal length of the glaſs bears no proportion to its diſtance; then will it be leaſt of all, as *C F* or *D F*; and the point *F* is call'd the focus of parallel rays. Any other focus, as *G*, being the focus of a near object, is call'd a reſpective focus; as it reſpects a particular diſtance: But the focus of parallel rays reſpects all objects that are at a very great diſtance; ſuch as is that of all the heavenly bodies.

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## Proposition II.

*The diſtance* HE *of the object from the glaſs, is to* EF, *the focal diſtance of parallel rays, as the diſtance* HG *of the object from its image is to* EG, *the diſtance of the image from the glaſs: That is,* HE : EF :: HG : EG.

The demonſtration of this propoſition may be gathered from any treatiſe of dioptrics; it being a general rule for finding the reſpective focus to any given diſtance, when the focus of parallel rays is known.

## Proposition III.

*The angle ſubtended by the diameter of the object, from the glaſs, is equal to that ſubtended, by the opening of the centres of the ſegments, from the focus of parallel rays. That is, the angle* AEB *equal to the angle* CFD.

### *Demonſtration.*

It appears, by inſpection of the figure, that $AB : CD :: HG : EG$.

And by the laſt propoſition $HE : EF :: HG : EG$.

Then, as the two laſt terms of theſe two analogies are alike; the two firſt erms of one will be in the ſame proportion as the two firſt terms of the other; which gives the following proportion: $AB : CD :: HE : EF$. Whence the truth of the propoſition is evident.

From

From this propoſition it appears, that the angle ſubtended by the diameter of the object from the glaſs, is found without any regard to the diſtance of the object, or to the diſtance of the reſpective focus, where the image is ſeen; as the meaſure depends entirely upon the focus of parallel rays and the opening of the ſegments. We may likewiſe, from hence, derive a rule for the quantity of the angle, without conſidering the length of the glaſs. Let an object, whoſe diameter is known, be ſet up at ſome known diſtance; the angle it will ſubtend from the glaſs may then be found by trigonometry: Then let it be meaſured by this micrometer, and the diſtance, between the centres of the ſegments, found on the ſcale already mentioned, will be the conſtant meaſure of the ſame angle, in all other caſes: Becauſe the diſtance of the object makes no alteration in the meaſure of the angle, as has been demonſtrated: And thus having obtain'd the diſtance between the centres of the ſegments, which anſwers to any one angle, all other diſtances may be computed by the rule of three.

All that has been hitherto ſaid relates to the firſt method of uſing this micrometer; that is, by fitting it to the end of a tube ſuited to its focal length, and by viewing the images with a proper eye-glaſs, in the manner of an aſtronomical teleſcope. But the length of the tube, in this way, would be very troubleſome; and therefore it will be proper to conſider other methods, for an eaſier management. I ſhall, therefore, proceed to the ſecond method, mention'd in *Obſ.* 10. which is, by uſing another object-glaſs to ſhorten the focus of that which ſerves for the micrometer. To

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facilitate the underſtanding of this method, it will be neceſſary to premiſe the following obſervation.

*Obſ.* 11. Rays of light, which are brought to ſuch convergency as to form the image of an object, proceed, after that, diverging, in the manner they did when they iſſued from the object before they were tranſmitted thro' the glaſs; and therefore they may be again collected by another ſpherical glaſs, ſo as to form a ſecond repreſentation of the ſame object; which may again be repeated by a third glaſs, *&c.* So that the firſt image may be conſider'd as an object to the ſecond glaſs and the ſecond image will be an object to the third, and ſo on. Tho' theſe images may be very different, in reſpect to their magnitudes, yet they will be all ſimilar; being true repreſentations of the ſame object: This will hold good, tho' the ſecond glaſs ſhould be put ſo near the firſt as to receive the rays before the image is form'd: For as the rays are tending to meet at a certain diſtance, the ſecond will receive them in that degree of convergency, and, by an additional refraction, bring them to a nearer focus; but the image will ſtill be ſimilar to that which would have been made by the firſt glaſs, if the ſecond had not been there.

Upon this principle all refracting teleſcopes are made; ſome of which are a combination of four, five, or ſix, glaſſes. The firſt glaſs forms an image of the object; the ſecond repeats the image, which it receives from the firſt; and ſo on, till the laſt glaſs brings a true repreſentation of the object to the eye. The ſame may be ſaid of reflecting teleſcopes: For a ſpherical mirror acts in the ſame manner, in that reſpect, as a ſpherical glaſs.

Now

Now let this be applied to the ſubject in hand. Suppoſe the focal diſtance of the divided object-glaſs to be about forty feet; and ſuppoſe the ſegments to be open'd wide enough to bring the oppoſite edges of an object in contact: Then let another object-glaſs, uncut, be fix'd within the tube, of a proper degree of convexity, to ſhorten the focus of the other as much as may be requir'd; ſuppoſe to twelve feet: By what has been juſt now obſerv'd, this glaſs will r preſent the two images in the ſame form which would have been exhibited by the divided glaſs, if this other glaſs had not been there. For tho' the images are not yet form'd, when the ſecond glaſs receives the rays; yet, as thoſe rays are converging towards it, the ſecond glaſs muſt repreſent thoſe images in the ſame poſition, and form, as the tendency of the rays requires. For while the ſegments are fix'd in their poſition to each other, their images will alſo be fix'd in their poſition; and let them be repeated ever ſo many times, by refraction thro' ſpherical glaſſes, or by reflexion from ſpherical mirrors, they can ſuffer no alteration in their poſition to one another. By this means, the teleſcope may be ſhorten'd, at pleaſure, tho' the ſcale for the meaſure of the angles will remain the ſame. The only inconvenience, which the ſhortneſs of the teleſcope introduces, is a want of ſufficient diſtinctneſs; which will ſo far hinder the exactneſs of the obſervation, as the contact of the edges cannot be ſo accurately determin'd, as they might be with longer teleſcopes.

This difficulty is intirely remov'd by fixing the divided glaſs at the end of a reflecting teleſcope: For the reflexions and refractions, which the rays muſt undergo

undergo in paſſing thro' the teleſcope, will no way alter the poſition of the images which the rays, that have paſſed thro' the ſegments, are tending to: For, as has been already obſerv'd, a number of reflexions and refractions may repeat the images, and alter their magnitudes; but can make no alteration in their proportions.

Therefore this way of fixing the divided glaſs to a reflecting teleſcope, which was the third method propos'd, is, by far, the beſt; as ſuch teleſcopes of moderate and manageable lengths, when well made, are capable of magnifying conſiderably, and ſhewing objects to great advantage. This micrometer's being applicable to the reflecting teleſcope, with ſo much certainty, is no inconſiderable advantage: For any one will eaſily underſtand, that, to meaſure the diameter of a planet exactly, it is neceſſary, that the planet be magnify'd, and ſhew'd diſtinctly, which could not be obtain'd, in the common way, without very great lengths; ſuch as render'd it very difficult, not to ſay impracticable, to take exact meaſures. Beſides, the common micrometer is limited, in this reſpect, upon another account; *viz.* becauſe the diameter of the planet cannot be meaſur'd, without having the whole planet within the field of the teleſcope which confines the magnifying power within very narrow bounds; whereas, by this method, nothing more is requir'd, than to ſee the contact of the edges, which allows the magnifying power to be increas'd at pleaſure.

In the common micrometer, the object is to be taken between two wires, ſo that the contact of its edges with thoſe wires cannot be obſerv'd at one view;

and the leaſt motion of the teleſcope, whilſt the obſerver is turning his eye from one wire to the other, muſt oblige him to repeat the obſervation; whereas, by this method the contact of the edges of the images is not at all affected by the motion of the teleſcope. Whence the compariſon of this micrometer, with the common ſort, in this reſpect, ſtands thus: The one requires great ſteadineſs in the teleſcope, but yet it is applicable to none, but ſuch as are very difficult to keep ſteady; the other does not require ſuch ſteadineſs, tho' it is applicable to ſhort teleſcopes, which are eaſily managed.

Theſe advantages not only add to the certainty of the obſervation, but aſſiſt vaſtly in the expedition; for an obſerver may make twenty obſervations, in this way, where he could ſcarce, with much fatigue, be ſure of one with the common micrometer. Expedition in making obſervations, muſt be allowed a very great advantage, in this climate, where the uncertainty of the weather renders aſtronomical obſervations ſo precarious, that no opportunities, even the moſt tranſient, ſhould be let ſlip. An inſtance of this was given in to the Royal Society, in an account of the eclipſe of the ſun laſt October.

As the motion of the teleſcope gives the obſerver no great inconvenience, in this method; neither does the motion of the object at all diſturb his obſervation (I mean ſuch a motion, as that of the heavens is). This gives him leave to take the diameter of a planet, in any direction; or the diſtance between two ſtars or planets, let their ſituation be how it will; in which reſpect the common micrometer is abſolutely defective; as it can give no angles, but ſuch as are perpendicular

pendicular to the line of their motion; tho' the diameters of the planets, in other directions, is very much wanted; it being highly probable, from the laws of motion, and what we ſee in Jupiter, that ſuch planets, as revolve round their axes, have their polar diameters ſhorter than their equatorial ones.

The diſtances of Jupiter's ſatellites from one another, or from Jupiter's body, cannot be meaſured, with any certainty, in the common way, as their poſition is always very far from being at right angles with the line of their motion: Neither can the moon's diameter, which muſt be taken from horn to horn, ſcarce ever be obtained that way, becauſe it very rarely happens, that the diameter, to be meaſured, lies at right angles to the line of her motion. The ſame may be ſaid of the diſtance between two ſtars. But this micrometer gives angles, in every direction, with equal eaſe and certainty; the obſervation being alſo finiſhed in an inſtant, without any trouble or fatigue to the obſerver. For as there are no wires made uſe of, this way, in the field of the teleſcope; ſo the obſerver has no concern about any illumination. The largeneſs of the ſcale deſerves alſo to be taken notice of, as it may, in this micrometer, be increas'd almoſt at pleaſure, according as the ſmalneſs of the object requires. Another inconvenience attending the common micrometer is, the variation of the ſcale, according to the diſtance of the object. As the teleſcope muſt be lengthen'd, or drawn out farther, for ſhort diſtances; the ſcale, which depends upon that length, is thereby increas'd; which renders the meaſure of the angle very uncertain: Whereas, in this micrometer, the ſcale is the ſame at all diſtances; ſo that the angle may be meaſured with the utmoſt cer-

tainty, without any regard to the diſtance of the object.

Upon the whole, it may be concluded, that this micrometer is a complete inſtrument in its kind; having many advantages above the common ſort, without any of their diſadvantages: And there is no doubt, but, when brought into practice, it will tend much to the advancement of aſtronomy.

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## LXXV. *An Account of an Earthquake felt at* York *on the* 19*th of* April 1754. *In a Letter from Mr.* David Erſkine Baker *to* Tho. Birch, *D. D. Secret. R. S.*

*Dear Sir,*

Read Apr. 25, 1754. ON Friday night laſt, the 19th inſtant, at about eleven o'clock, we were alarm'd, in this city, with the ſhock of an earthquake. As I was myſelf in London, and felt both the ſhocks, which happen'd there in the year 1750, I became immediately ſenſible of what it was. In the room, in which I was ſitting, which was on a firſt floor, the tremulous, or rather undulating, motion of the floor was very plain; and the windows rattled, as if they had been ſhaken by a ſudden ſquall of wind. The ſhock laſted for about three ſeconds, and was attended, or rather preceded, by a rumbling noiſe, not much unlike that made by an empty hearſe driven over a ſtone pavement, and, indeed, exactly the ſame with that, which I remember to have heard, with both the ſhocks, in London, in 1750. The violence of the ſhock